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Generation of water waves by turbulent wind flow

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GENERATION OF WATER WAVES
BY TURBULENT WIND FLOW

by

Paul Richard Klinedinst, Jr.

UNITED STATES
NAVAL POSTGRADUATE SCHOOL

THESIS

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June 1968

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requirements for the degree of

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ABSTRACT

The paper is concerned with a theoretical study of shearing flow

bounded by a wavy surface with consideration of turbulent flow above

the air-sea interface. Account is to be taken of turbulence in the flow

through the use of Reynolds stresses associated with turbulent flow.

An adaptation of the mixing length theory as applied to pipe flow is made

for channel flow and the resulting mixing length versus energy relation-

ship is incorporated in the Reynolds stress term. Finally, the rate of

wave growth is calculated from the normal surface pressure in phase

with the wave slope.

A curvilinear coordinate system which follows with the wave train

is used in order to simplify the formulation of the problem. All para-

meters are non-dimensionalized and the analysis is made considering

a velocity profile adapted from pipe to channel flow.

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SYMBOLS AND ABBREVIATIONS

a   non-dimensional wave amplitude

A   constant parameter

c   non-dimensional wave celerity

C   constant

d   position at which upper boundary conditions are applied

E'   turbulent energy

E   wave energy

$\bar{\dot{E}}$ rate of energy input to wave

F   function determined by solution

$\mathbb{F}$ Froude number

g   acceleration of gravity (980 cm sec^{-2})

H   channel height

J   Jacobian

K   kinematic eddy viscosity

ℓ mixing length

ℓ_1 mixing length (perturbation term)

L   mixing length (zero-order term)

P   pressure

q   velocity of main flow above boundary layer

U   velocity of main flow in the channel

u   non-dimensional velocity parallel to ξ axis

u_* friction velocity

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v	non-dimensional velocity parallel to η axis
α	constant
χ	real component of pressure
δ	imaginary component of pressure
ϵ	real component of shear stress
ε	imaginary component of shear stress
γ	constant parameter
κ	mixing length constant (0.4)
λ	non-dimensional wave length
ψ	stream function
μ	molecular viscosity
ρ_a	air density (0.0013 g cm^{-3})
ρ_w	water density (1.027 g cm^{-3})
τ	shear stress
ξ, η	curvilinear coordinates

ACKNOWLEDGMENT

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1. INTRODUCTION

A semi-empirical study of turbulent air flow over a wavy surface is made in which normal and tangential stresses on the surface are examined with the object of estimating the growth of water waves due to a turbulent wind.

Ursell (1956) gives a summary and a critical review of the theories of wind-wave formation extant in 1956. The review gave the necessary impetus for further investigations, both theoretical and experimental, in this area. Since 1956 there has been much work done on the generation of water waves by surface stresses induced by the wind. Miles (1957) and Benjamin (1959) considered laminar air flow over a wavy water surface and deduced formulae for exponential wave growth due to an instability mechanism. Later Phillips (1966) included air-stream turbulence in the instability model from a semi-empirical point of view.

In his initial work, Miles (1957) assumes an inviscid, incompressible, two-dimensional, parallel shear flow coupled with a prescribed two-dimensional deep-water gravity wave. Turbulent fluctuations in the perturbation equations of motion are neglected, even though they are decisive in maintaining the mean shear flow. Furthermore, the wave-induced velocities and pressure are small, which justifies linearization of the equations. The water is also assumed to be inviscid, incompressible, and irrotational, and the slope of the displaced surface is assumed to be small. Only that component of the aerodynamic force in phase

with the wave slope is important in the wave-generation mechanism. The viscous drag forces of the air are assumed to be negligible compared with the normal pressure in their effect on the surface wave.

This model is used to determine a first-order approximation to the disturbed motion of the air and the consequent energy transfer to the wave. The velocity distribution is taken to be that for turbulent flow over an aerodynamically smooth water surface. The result rests on the solution of the inviscid Orr-Sommerfeld equation which has a singularity at the height where $U = c$ (the critical level). An exponential increase in amplitude of water waves with phase velocity c occurs if the mean air flow velocity $U(y)$ in the direction of wave travel varies with height above the surface such that U'' at the critical level is negative. This is almost always the case.

Phillips (1966) attempts to include wave-induced turbulence in the instability model by including in the momentum flux equation an appropriate integral over the entire layer of air of the contribution due to perturbation eddy viscosities. This integral is multiplied by an unknown constant A which is estimated from experimental results from flow over fixed wavy surfaces. The uncertainty in A is estimated to be $\pm$ 50 per cent. Phillips concludes that even though there exists a high degree of uncertainty in A the results do provide a framework in which experimental measurements may be interpreted.

According to Miles (1967), experimental results indicate that a theoretical model based on laminar flow may be adequate in the

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laboratory but not on an oceanographic scale. The inviscid, laminar model underestimates the energy transfer from wind to waves over at least a significant portion of the spectrum for an open sea. This suggests that the significance of wave-induced perturbations in the turbulent Reynolds stresses for momentum and energy transfer from wind to wave must increase with an appropriate scale factor. Miles concludes that further theoretical work in this area appears to demand some 'ad hoc' hypothesis to describe these perturbation Reynolds stresses.

This can be done by considering a slightly generalized form of the classical mixing-length hypothesis. A means must be found to predict the value for the change in mixing length upon which the perturbation Reynolds stresses are based. Zagustin (1963) has given a semi-empirical solution for turbulent flow in a pipe which gives good agreement with experimental data and which predicts the value of the mixing length. This model is generalized below to include flow over a wavy surface in a manner analogous to that of Miles and Benjamin in that it assumes the flow to follow the wavy surface. Viscous stress terms are replaced by the Reynolds stresses associated with turbulent flow. Normal Reynolds stresses are neglected. Zagustin's method of closing the system of mean turbulent equations of motion is not unique (see, for example, Squire, 1959), but there seems to be no good reason to use any other model in its place. The investigation below details the application of Zagustin's work to this model and outlines the solution of the differential equations

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using numerical methods and appropriate boundary conditions. The

unknown parameters in the model are fitted to experimental data describing

the normal pressure in phase with the wave slope.

2. FORMULATION OF THE MODEL

The zero-order flow is taken as flow in a two-dimensional channel.
All lengths are made non-dimensional using the channel halfwidth, H.
Velocities are made non-dimensional using the midstream velocity, U_0 .
All stresses are made non-dimensional by dividing by $\rho_a U_0^2$, where ρ_a
is the density of air.

2.1 THE ZERO-ORDER FLOW

The velocity is expressed in terms of a stationary and a fluctuating
(turbulent) part:

$$(2.1) \qquad u = U + u'$$

Following Zagustin, the Reynolds shear stresses τ are written as

$$(2.2) \qquad \tau = L^2 (grad\, q)^2$$

where q is the mean speed. This can be considered a defining
equation for L (although here it appears as the usual mixing length)
in that L will not be prescribed, but will be part of the solution. Then
the two-dimensional mean equation of motion is

$$(2.3) \qquad (U \cdot grad)\, U = -grad\, P + \left| \hat{k} \times grad\, \tau \right|$$

where $\hat{k}$ is a unit vector normal to the plane of the flow, P is the
pressure, and $K = L^2\, grad\, q$ is the eddy viscosity. The turbulent

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kinetic energy, $E' = \vec{u}' \cdot \vec{u}'$, is assumed to have the form

$$(2.4) \qquad E' = C \, L^2 (\text{grad } q)^2$$

where C is an unknown constant. It is assumed that the rate of
dissipation of E' is proportional to E'.

A flux of turbulent kinetic energy into any volume element is assumed
to occur when the quantity L varies over the element. Thus, the
rate of inflow of E' is

$$(2.5) \qquad \propto \iint_S (\text{grad } L \, , \, \hat{n}) \, dS$$

where $\hat{n}$ is the normal to any surface and $\propto$ is an unknown constant
depending on the turbulent intensity. The turbulent kinetic energy is
constant at any point, and the above assumptions then imply that

$$(2.6) \qquad \iint_S (\text{grad } L \, , \, \hat{n}) \, dS + B \iiint_V E' \, dV = 0,$$

where B is an unknown constant. Applying the divergence theorem,
considering V to be an arbitrary volume, and using equation (2.4),
equation (2.6) becomes

$$(2.7) \qquad \nabla^2 L + B \, L^2 (\text{grad } q)^2 = 0.$$

Equations (2.3) and (2.7), together with the continuity equation,
form a closed set for the mean velocities and the quantity L . The
constant B must be determined experimentally.

For flow between two parallel walls as shown in Figure 1, the continuity equation is trivial, and equations (2.3) and (2.7) become

$$(2.8) \qquad -\text{grad } P + \frac{d}{dy}\left[L^2 \left(d\frac{U}{dy}\right)^2 \right] = 0 \qquad \text{and}$$

$$(2.9) \qquad \frac{d^2 L}{dy^2} + B\, L^2 \left(d\frac{U}{dy}\right)^2 = 0.$$

$y = 1$

$y = -1$
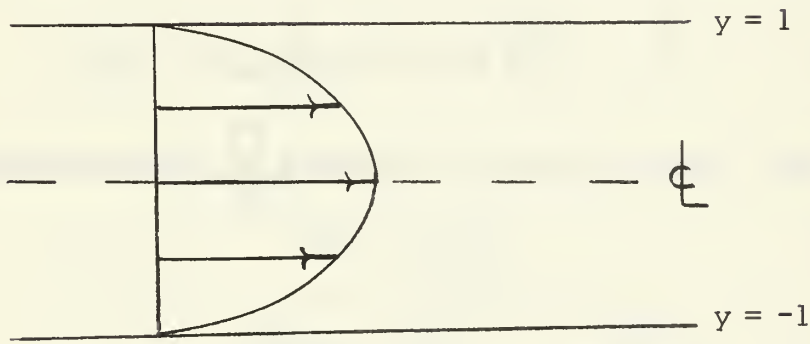
Figure 1.

Defining $B = \dfrac{A}{2}$ for the zero-order flow case and setting $\lambda = |\text{grad } P|$, equation (2.9) becomes

$$(2.10) \qquad \frac{d^2 L}{dy^2} + \frac{A}{2}\, L^2 \left(d\frac{U}{dy}\right)^2 = 0,$$

and equation (2.9), after integrating, becomes

$$(2.11) \qquad hy = L^2 \left(d\frac{U}{dy}\right)^2 .$$

Equations (2.10) and (2.11) give

$$(2.12) \qquad \frac{d^2 L}{dy^2} = -\frac{A}{2}\, hy,$$

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which has the boundary conditions:

$$\frac{dL}{dy}\bigg|_{y=0} = 0 \quad \text{and} \quad L\bigg|_{y=1} = L_o,$$

where L_o is a quantity analogous to the roughness length in mixing-length theory. Integrating equation (2.12) twice and applying these boundary conditions results in

$$(2.13) \qquad L = \frac{Ah}{12}(1 - y^3) + L_o.$$

Next, solving equation (2.11) for $\frac{dU}{dy}$ and substituting for L gives

$$(2.14) \qquad \frac{dU}{dy} = - \frac{\sqrt{hy}}{L_o + \frac{Ah}{12}(1 - y^3)}.$$

By defining

$$b = 1 + \frac{12 L_o}{Ah} \qquad \text{and}$$

$$z = \frac{y}{\sqrt[3]{b}},$$

equation (2.14) simplifies to

$$(2.15) \qquad \left(\frac{12}{A\sqrt{hb}}\right)^{-1} dU = - \frac{\sqrt{z}}{1 - z^3} dz.$$

Integrating equation (2.15), applying the no-slip condition at $z = -\sqrt[3]{b}$, and returning to the y coordinate gives

$$(2.16) \qquad U_{(y)} = \frac{8}{A\sqrt{hb}} \left[\tanh^{-1}(\frac{1}{\sqrt{b}}) - \tanh^{-1}(\frac{y^{3/2}}{\sqrt{b}}) \right].$$

The mean velocity at mid-channel is

$$U_{(y)} \Big|_{y=0} = 1$$

which, when applied to equation (2.16), results in the expression

$$(2.17) \qquad \frac{8}{A\sqrt{hb}} \tanh^{-1}(\frac{1}{\sqrt{b}}) = 1.$$

Equation (2.16) therefore becomes

$$(2.18) \qquad U_{(y)} = 1 - \frac{\tanh^{-1}(\frac{y^{3/2}}{\sqrt{b}})}{\tanh^{-1}(\frac{1}{\sqrt{b}})}.$$

The expression (2.13) for L can be simplified by recalling that von Karman's constant, K , gives the rate of growth of L at the wall. Therefore,

$$(2.19) \qquad K = \frac{dL}{dy} \Big|_{y=-1}$$

where $K \approx 0.4$ is the accepted value. Then, from equations (2.13) it is found that

$$(2.20) \qquad Ah = 1.6$$

$$(2.21) \qquad b = 1 + 7.5\, L_o$$

$$(2.22) \qquad L = \frac{b - y^3}{7.5}.$$

Following Benjamin (1959), the equations of motion in terms of ξ and η are

$$(2.30a) \qquad J(\psi_\eta \psi_{\eta\xi} - \psi_\xi \psi_{\eta\eta}) + 1/2 J_\xi(\psi_\xi^2 + \psi_\eta^2) = -P_\xi + \tau_\eta$$

$$(b) \qquad J(-\psi_\eta \psi_{\xi\xi} + \psi_\xi \psi_{\xi\eta}) + 1/2 J_\eta(\psi_\xi^2 + \psi_\eta^2) = -P_\eta + \tau_\xi$$

where the subscripts denote partial differentiation.

The stream function is composed of a zero-order term plus a periodic perturbation. When there is no wave, the stream function is

$$(2.31) \qquad \psi_o = \int_o^\eta \left[U(\eta) - c \right] d\eta .$$

where $\eta + y = 1$.

$U(\eta)$ is the velocity profile from equation (2.18) expressed in curvilinear coordinates

$$(2.32) \qquad U(\eta) = 1 - \frac{\tanh^{-1}\left[\frac{1}{\sqrt{b}}(1-\eta)^{3/2} \right]}{\tanh^{-1}\left(\frac{1}{\sqrt{b}} \right)} .$$

When the water surface is disturbed by a stationary wave, the stream function becomes

$$(2.33) \qquad \psi(\xi,\eta) = \psi_o + a\left\{ F(\eta) + \left[U(\eta) - c \right] e^{-k\eta} \right\} e^{ik\xi}$$

where $F(\eta)$ is to be determined.

The mixing length is expressed in a form similar to that for ψ :

$$(2.34) \qquad \ell = L(\eta) + a \ell_i(\eta) e^{ik\xi}$$

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where $L(\eta)$ is given above and $\ell_1(\eta)$ is to be determined. Similarly,

(2.35)
$$B = \frac{A}{2} + a\,\delta\,e^{ik\xi}$$

where A is given above and δ is an unknown parameter to be determined empirically. The velocity components parallel to ξ and η are given by

(2.36a)
$$u = J^{1/2}\psi_\eta = U - c + a\left\{F'(\eta) + U'(\eta)e^{-k\eta}\right\}e^{ik\xi}$$

b)
$$v = J^{1/2}\psi_\xi = -ika\left\{F(\eta) + [U(\eta) - c]e^{-k\eta}\right\}e^{ik\xi}.$$

To formulate the differential equation in ℓ_1 to be solved, substitution is made for ℓ, B and q in equation (2.7) expressed in curvilinear coordinates:

(2.37)
$$\nabla^2\ell + B\left(\frac{dq}{d\eta}\right)^2 = 0.$$

Simplification by elimination of all terms of $0(a^2)$ and above yields

(2.38)
$$\ell_1'' + \ell_1(AE - k^2) + AHF'' + \delta D +$$
$$Ae^{-k\eta}(G - kD) = 0$$

where

$$D = U'^2 L^2 \qquad \text{and}$$

$$E = U'^2 L.$$

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Similarly, when equations (2.30) are expanded, cross differentiated to eliminate pressure terms, and linearized in a, one obtains

$$(2.39) \qquad \frac{ik}{2} \left\{ (U - c)(F'' - k^2 F) - U'' F \right\} \, \text{sgn}(U') =$$

$$(\frac{d^2}{d\eta^2} + k^2)(HF'' + E\ell_1) +$$

$$e^{-k\eta} (G'' - 2kG' + 2k^2 G)$$

where

$$G = U'U''L^2$$
$$H = U'L^2$$

and where U' is positive for the lower half of the channel. Note that a singularity occurs at mid-channel ($U' = 0$). Equations (2.38) and (2.39) are to be solved for F and ℓ_1.

The boundary conditions are as follows. At the interface where $\eta = 0$, $U(0) = 0$ to satisfy the no-slip condition, and $v = 0$ because the wave is stationary. Therefore, from equation (2.36b):

$$(2.40) \qquad F(0) = c.$$

The velocities in the water must take on negative values in order that the shearing stress across the interface be continuous. The variable component of the tangential water velocity associated with an irrotational wave is of the form

$$(2.41) \qquad u' = a \sqrt{k} \sqrt{\frac{gH}{U_o^2}} \, e^{ik\xi} .$$

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The total tangential water velocity is

$$(2.42) \qquad u = -c - a \sqrt{k} \sqrt{\frac{gH}{U_o^2}} \ e^{ik\xi} .$$

Defining the Froude number:

$$(2.43) \qquad \mathbb{F} = \frac{U_o^2}{gH} , \text{ and invoking the no-slip condition, the}$$

tangential air velocity is

$$(2.44) \qquad u = -c - a \sqrt{\frac{k}{\mathbb{F}}} \ e^{ik\xi} \qquad \text{at } \eta = 0, \text{ or}$$

$$(2.45) \qquad u = -c - a \beta \ e^{ik\xi} \qquad \text{at } \eta = 0$$

where $\beta = \sqrt{\frac{k}{\mathbb{F}}}$. From equation (2.36a) the second lower boundary
condition then becomes

$$(2.46) \qquad F'(0) = - U'(0) + \beta .$$

The perturbation in ℓ at the water surface, $\ell_1(0)$, must also be
fixed. This must be regarded (along with δ) as an unknown parameter
in the problem, to be adjusted empirically.

The perturbed flow must die away at some large distance (relative
to k^{-1}) from the wavy surface. Thus, as η becomes large,

$$(2.47) \qquad F(\eta) \longrightarrow (c - U) e^{-k\eta}$$

$$(2.48) \qquad F'(\eta) \longrightarrow - U' e^{-k\eta}$$

$$(2.49) \qquad \ell_1(\eta) \longrightarrow 0 .$$

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2.3 NORMAL PRESSURE, SHEAR STRESS AND WAVE GROWTH

The pressure on the wavy surface is the sum of that due to normal atmospheric turbulence and that generated by air moving over the undulating surface. The latter pressure predominates in the generation mechanism considered here and is expressed as

$$(2.50) \qquad P(0) = a(\chi + i\delta)e^{ik\xi}$$

where χ and δ are to be determined. Differentiating equation (2.50) with respect to ξ, substituting the result into equation (2.30a), and simplifying yields

$$(2.51) \qquad P(0) = \left[U'F - (U - c)F' - \frac{2i}{k} \frac{d}{d\eta}(HF'' + E\ell_i) + (G' - Gk) \right]_{\eta = 0}$$

which gives χ and δ.

In the same manner, the shear stress at the wall is expressed as

$$(2.52) \qquad \tau(0) = a(\epsilon + i\varepsilon)e^{ik\xi}$$

where ϵ and ε are to be determined. To evaluate the shear stress, equation (2.2) is expanded in terms of ψ :

$$(2.53) \qquad \tau = \ell^2 J^2 \psi_{\eta\eta}^2 + J_\eta \psi_\eta \psi_{\eta\eta}).$$

Substituting for ℓ, J, ψ, or their derivatives where required obtains

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$$(2.54) \qquad \tau(0) = \left(2 \lfloor U' \lfloor U' \; + \; \lfloor (F'' + U'') \rfloor \right)_{\eta = 0}$$

which gives ϵ and ε. The shear stress seems to be relatively

unimportant to wave growth, however, and will not be considered herein.

To compute the wave growth rate the normal pressure in phase with

the wave slope is required. Expansion of equation (2.50) and comparison

with equation (2.25) for the wave form show that a positive δ results

in wave growth. The average energy input due to normal pressure is

$$(2.55) \qquad \bar{\dot{E}} = -\frac{1}{\lambda} \int_{0}^{\lambda} P \frac{d\eta}{dt} \, d\xi$$

or, in dimensional terms,

$$(2.56) \qquad \bar{\dot{E}} = \frac{a^2}{2H} \omega \rho_a U_o^{2} \delta,$$

where $\omega^2 = gk$ is the wave frequency.

The surface energy of the wave is

$$(2.57) \qquad E = 1/2 \, \rho_w \, ga^2$$

where ρ_w is the water density, so that

$$(2.58) \qquad \bar{\dot{E}} = \rho_w \, ga\frac{da}{dt} \, .$$

Equating equations (2.56) and (2.58) and solving for $\frac{da}{dt}$ gives

$$(2.59) \qquad \frac{da}{dt} = a\frac{\rho_a}{\rho_w} \frac{U_o^{2}}{2gH} \delta \omega \, .$$

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Integrating produces the familiar exponential wave growth equation:

$$(2.60) \qquad a = a_o \exp \left\{ \frac{\rho_a}{\rho_w} \quad \frac{F}{2} \delta \omega t \right\} .$$

The rate of energy input due to surface shear is

$$(2.61) \qquad \bar{\dot{E}} = \frac{1}{\lambda} \int_0^\lambda u \, \tau \, d\xi$$

or

$$(2.62) \qquad \bar{\dot{E}} = a^2 \rho_a \frac{U_o^2}{2H} \beta \epsilon .$$

This energy is expected to go, at least initially, into a surface shear flow.

3. SOLUTION OF THE FIRST-ORDER PROBLEM

A solution for the function F and its derivatives must now be found so that values for normal pressure, shear stress and wave growth rate may be determined. There are several numerical integration methods available for this purpose, having the common property of starting at one boundary and ending at the other. However, the problem as formulated has conditions at both boundaries, and a method of super-position of solutions must be used.

Equations (2.38) and (2.39) can be rewritten as six linear first-order ordinary differential equations. That is letting:

(3.1a) $F = Y_1$

    b) $F' = Y_2$

    c) $F'' = Y_3$

    d) $F''' = Y_4$

    e) $\ell_1 = Y_5$

    f) $\ell_1' = Y_6$

gives six ordinary differential equations:

(3.2a) $Y_1' = Y_2$

    b) $Y_2' = Y_3$

    c) $Y_3' = Y_4$

d) $\quad Y'_4 = \dfrac{1}{H}\left\{ \dfrac{ik}{2}\left[(U - c)(Y_3 - k^2 Y_1) - U'' Y_1 \right] - 2H'Y_4 \right.$

$\qquad\qquad - (H'' + k^2 H)Y_3 - E\ell'' - 2E'Y_6 - (E'' + k^2 E)Y_5$

$\qquad\qquad \left. - e^{-k\eta}(2Gk^2 - 2G'k + G'') \right\}$

and

e) $\quad Y'_5 = Y_6$

f) $\quad Y'_6 = -Y_5(AE - k^2) - AHY_3 - \gamma D$

$\qquad\qquad -Ae^{-k\eta}(G - kD).$

There are now six equations, each requiring an initial input at the lower channel boundary. Linearly independent solutions are found by starting repeatedly at the lower boundary with different initial values, Y(0). The final solution is obtained by superposing the independent solutions to fit the required conditions at the upper boundary.

The independent solutions, denoted by the superscript j, have initial values given by

(3.3) $\qquad Y_i^{j+1}(0) = \delta_i^{\,j}$ at $\eta = 0$,

where δ is the Kronecker delta. The solution to the homogeneous form of equations (3.2) can be written as a sum of independent solutions to the homogeneous equation:

(3.4) $\qquad Y_i = \displaystyle\sum_{j=1}^{6} J_j Y_i^j$

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where the J's are complex constants. Since

$$F(0) = F'(0) = \ell_1(0) = 0$$

for the homogeneous system, the constants J_1, J_2, and J_5 in equation (3.4) must equal zero. The complete solution is achieved by adding to the homogeneous solution a particular solution of equation (3.2). For this particular solution, the initial conditions are

(3.5a) $Y_1^p = c$

  b) $Y_2^p = \beta - U'$

  c) $Y_3^p = 0$

  d) $Y_4^p = 0$

  e) $Y_5^p = \ell_1(0)$

  f) $Y_6^p = 0.$

Then the complete solution, satisfying all the boundary conditions at $\eta = 0$, is

(3.6) $Y_i(\eta) = J_3 Y_i^3(\eta) + J_4 Y_i^4(\eta) + J_6 Y_i^6(\eta) + Y_i^p(\eta),$

where the J's are to be determined.

The position of the upper boundary, where conditions (2.47), (2.48), and (2.49) hold, is not known, and must be determined by a convergence criterion. The solution is determined by applying the upper boundary condition at various increasing values of $\eta = d$, solving the resulting equations (3.6) evaluated at d for the J's, and evaluating the wall pressure each time using equation (2.51). When the wall pressure determined in this manner converges, the procedure is stopped.

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4. THE COMPUTER SOLUTION

The solution for the function F and its derivatives has been obtained by solving the problem as outlined in section 3 on an IBM 360-67 computer using complex double-precision arithmetic. Major sub-routines are incorporated in the program to calculate the velocity profile, perform the integration using the Runge-Kutta Gill fourth-order method, compute the shear stress, calculate the wave growth rate and evaluate the stream function. Appendix I contains the format of the program and a set of results obtained.

The accuracy of the Runge-Kutta Gill integration method depends on the size of the step used in the iteration. The value 10^{-3} was selected in order to determine whether the surface pressure converged on a particular value. Because surface pressure did not converge for this step size, it was decreased until convergence was achieved. A step size of 10^{-6} was necessary to obtain convergence of the wall pressure, and the program running time was about 112 minutes. A final optimum step size of 10^{-5} for η less than 0.3 and 2.5×10^{-5} for η greater than 0.3 was then adopted. This step size gives convergence on a result essentially the same as that for 10^{-6} and a running time of 65 minutes.

5. RESULTS AND CONCLUSIONS

The zero-order velocity profile is given by equation (2.18) which has $L(0)$ as a parameter. A plot of $U(y)$ for several values of $L(0)$ appears in Figure 3. From the figure it can be seen that as the parameter $L(0)$ decreases, the shape of the profile changes from nearly linear to almost a step function. A representative logarithmic profile is also shown in the same figure so that the two profile shapes may be easily compared.
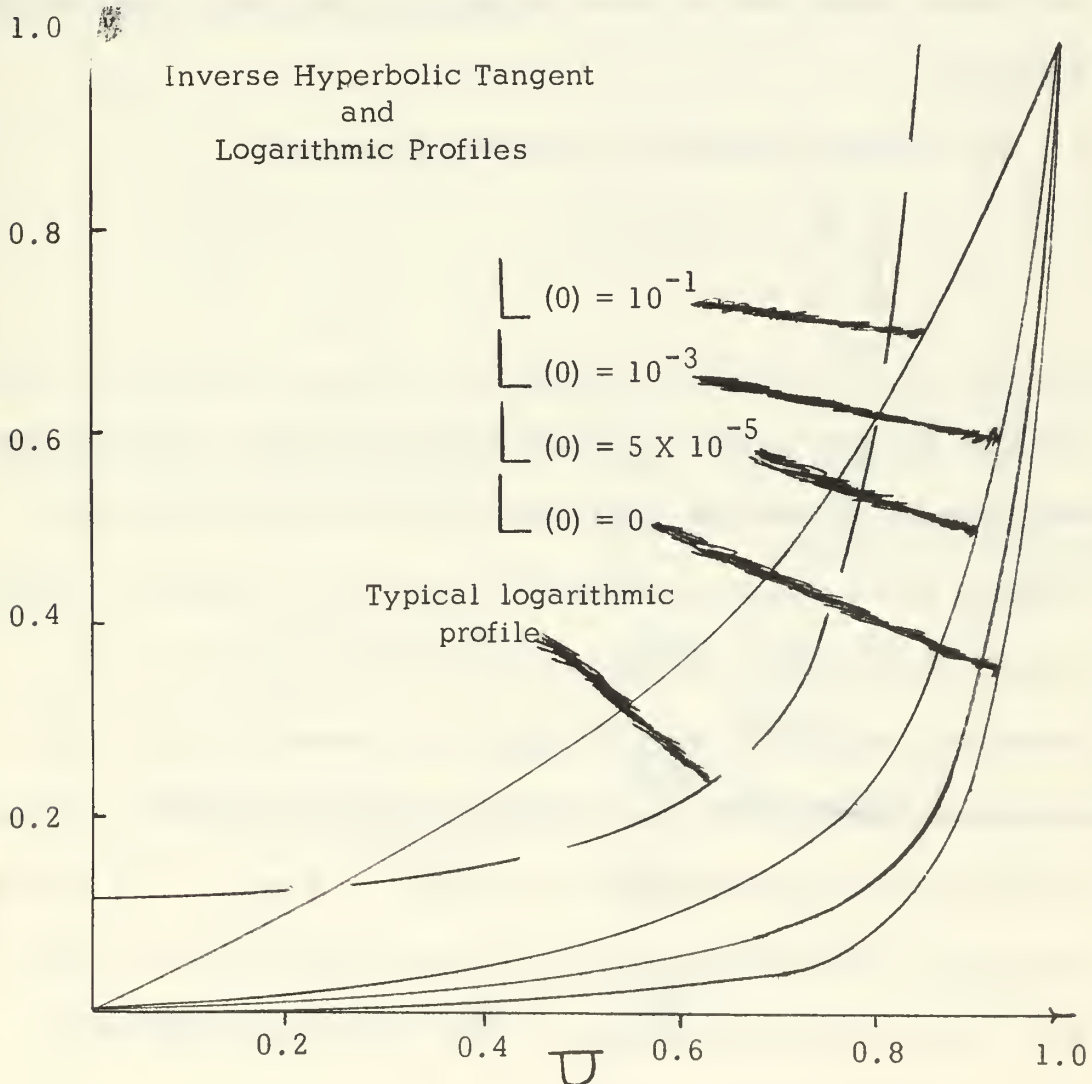
Figure 3.

The remainder of the analysis is concerned with the determination

of suitable values for the parameters $L(0)$, $\ell_1(0)$, and δ . For

the channel under consideration:

$$H = 10 \text{ meters}$$

$$U_o = 10 \text{ meters/second}$$

which results in a Froude number of 1.018. For the wave number,

$$k = 15,$$

the instability mechanism considered is important. These values for

the Froude number and the wave number have been used throughout the

analysis.

The following parameter values were first used:

$$L(0) = 2.22 \times 10^{-2}$$

$$\ell_1(0) = 0$$

$$\delta = 10.$$

The wall pressure failed to converge for these values. Convergence

was assumed to have resulted if wall pressure values were nearly

constant over a significant range of d (say 0.1) or reached a minimum

value at some point in the channel. (This latter criterion is not

completely defensible, but the singularity in equation (3.2d) at $\eta = 1$,

itself a peculiarity of the Zagustin assumptions, is probably responsible

for the erratic behavior of the wall pressure as $\eta \rightarrow 1$.) δ was then

varied from -1000 through zero to +1000 and given imaginary values as

well. The wall pressure changed only in the fourth or fifth decimal

which was considered insignificant, although ℓ_1 changed markedly.

Thus, the solution seems insensitive to δ.

New values for the parameters were then used:

$$L(0) = 5 \times 10^{-5}$$
$$\ell_1(0) = 0$$
$$\delta = 10.$$

The wall pressure converged for these values, but the value was too

high in magnitude and of the wrong sign. A value nearer to 5 (Miles,

1957) was considered appropriate.

With

$$L(0) = 5 \times 10^{-5}$$
$$\ell_1(0) = 1$$
$$\delta = 10$$

the resulting wall pressure was positive but still too high in magnitude.

Figure 4 summarizes the results of varying $L(0)$ and $\ell_1(0)$.
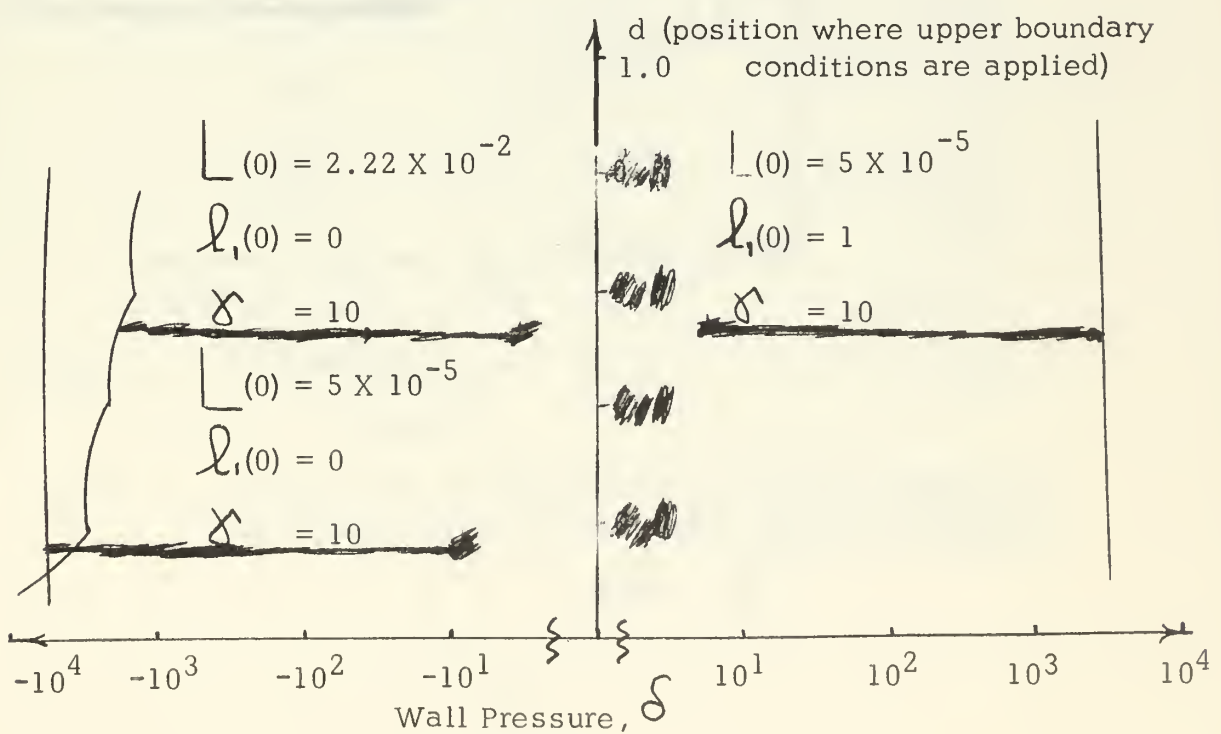
Figure 4.

It appeared that the wall pressure was more directly dependent on $\ell_1(0)$ than on any other parameter. Subsequent investigation showed that for any η the wall pressure was a linear function of the real or imaginary part of $\ell_1(0)$ providing that the other component was held constant. For

$$L(0) = 5 \times 10^{-5}$$
$$\ell_1(0) = 0.65788$$
$$\gamma = 10$$

the resulting minimum wall pressure was 3.83 at d = 0.48. Figure 5 shows the resulting wall pressure in this case versus d.
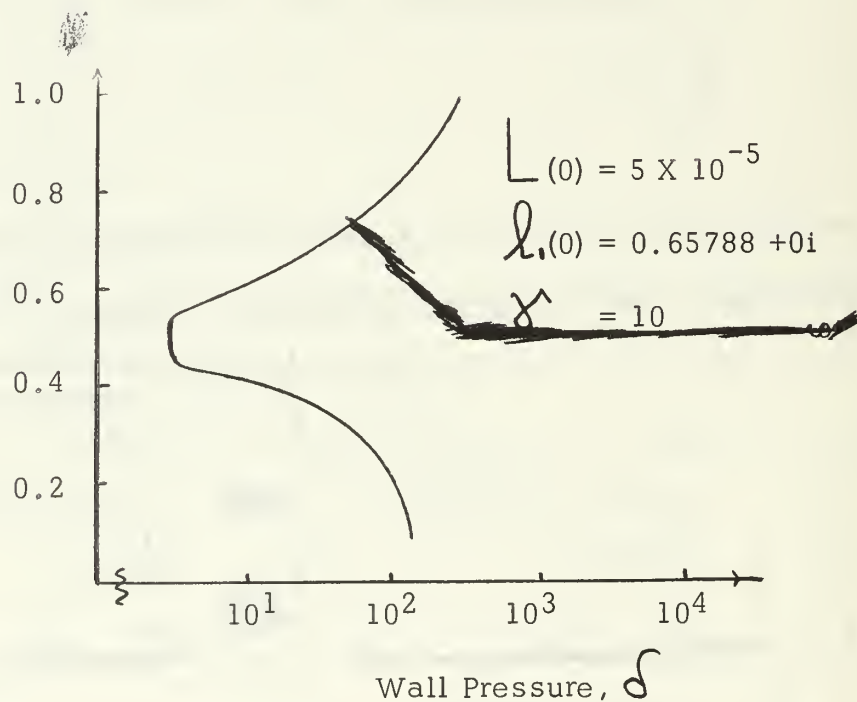
Figure 5.

γ was then given a value:

$$\gamma = 1000.$$

No significant change occurred in the wall pressure. Convergence, as defined above, always occurred near d = 0.5.

Table 1 summarizes the values of the wall pressure, shear stress, and wave growth rate versus $\ell_1(0)$ at d = 0.5 for parameter values:

$$L(0) = 5 \times 10^{-5}$$
$$\gamma = 10.$$

$\ell_1(0)$	Out-of-phase Wall Pressure, δ	In-phase Shear Stress, ϵ	Wave Growth Rate, $\frac{\rho_a}{\rho_w} \frac{E}{2} \delta\omega$
0.0 + 0i	-6495.3	-29.78	-1.61
0.2 + 0i	-4519.4	-14.89	-1.12
0.4 + 0i	-2543.6	0.001	-0.63
0.6 + 0i	-567.7	14.87	-0.141
0.8 + 0i	1408.2	29.76	0.349
1.0 + 0i	3384.0	44.64	0.838
0.65788+0i	3.83	19.19	0.001
0.0 + 0.2i	-6495.3	-29.78	-1.61
0.0 + 0.4i	-6401.8	-29.78	-1.58
0.0 + 0.6i	-6308.3	-29.78	-1.55
0.0 + 0.8i	-6214.8	-29.78	-1.52
0.0 + 1.0i	-6221.3	-29.78	-1.49
0.65788+0.1i	50.9	-29.78	0.013

Table 1. Wall pressure, shear stress, and wave growth rate as a function of the perturbation surface roughness.

It can be seen that imaginary values for $\mathcal{L}_1(0)$ have no effect on the shear stress. The required value for $\mathcal{L}_1(0)$ is $0.65788 + 0i$ for a wall pressure of 3.83, but the shear stress seems high. (However, the shear stress value is a first-order contribution to the total shear stress and is reduced by the fractional value of the wave amplitude.) Exponential wave growth rates, due to the wall pressure alone, appear reasonable. To reduce the shear stress to a more acceptable value in relation to the normal pressure, parameter values

$$L(0) = 5 \times 10^{-5}$$
$$\mathcal{L}_1(0) = 0.4 + 5.45i$$
$$\gamma = 10$$

were used, and give at $\eta = 0.5$

$$\text{Normal pressure} = 5.0$$
$$\text{Shear stress} = 0.001$$
$$\text{Growth rate} = 0.001.$$

However, the wall pressure failed to converge, and had the form shown in Figure 6.

Equation (2.33) for the stream function has been evaluated with the values of F calculated from the constants J determined at d = 0.48 where the wall pressure was found to converge. Parameter values were

$$L(0) = 5 \times 10^{-5}$$
$$\mathcal{L}_1(0) = 0.65788$$
$$\gamma = 10.$$

Figure 6.

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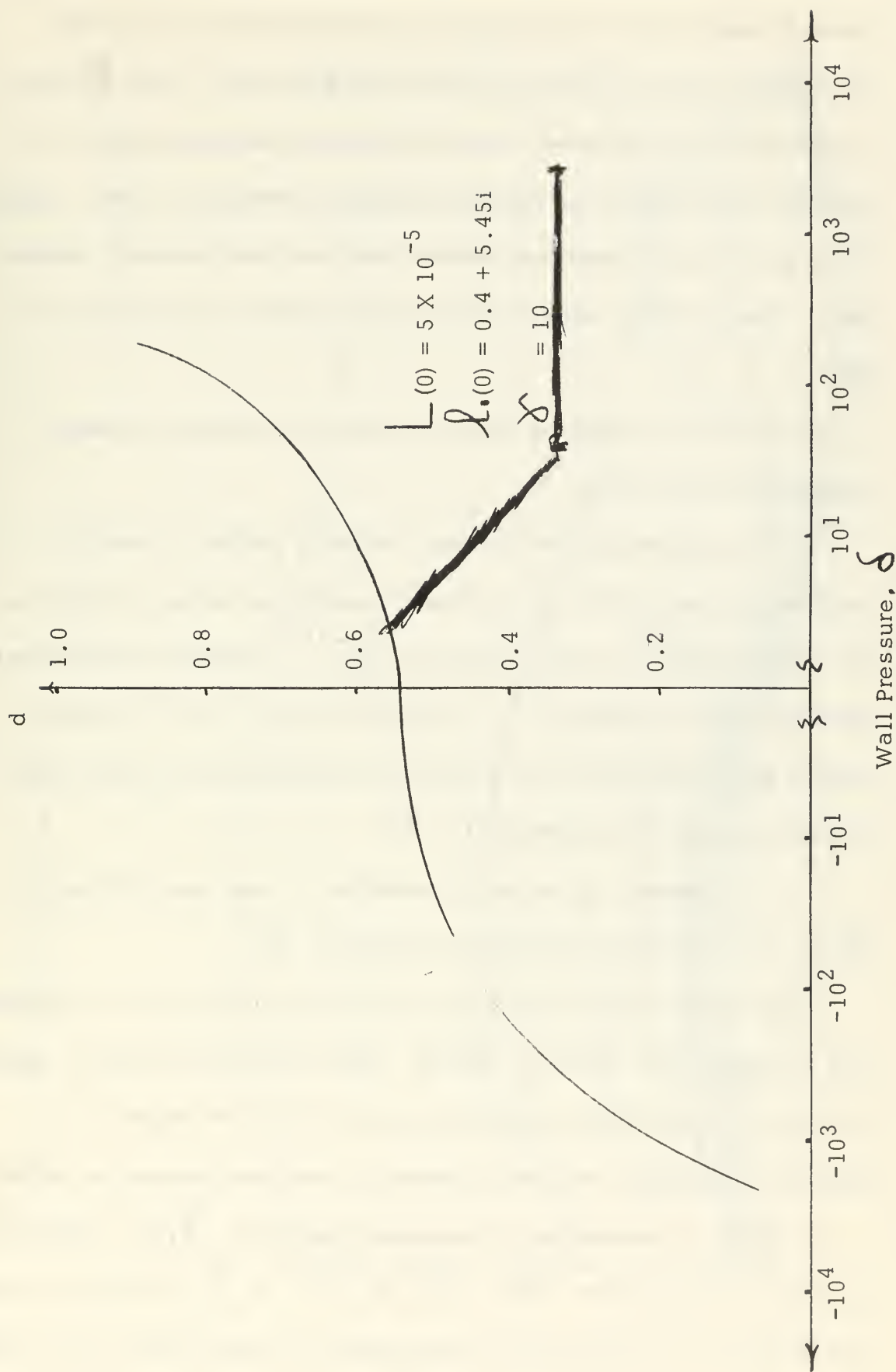
Figure 7 shows a plot of the streamlines with the 'cats eye' present and centered about 30 degrees ahead of the wave crest. The 'cats eye' or closed loop is indicative of the wave induced perturbations in the flow and characterizes the region of turbulence (Phillips, 1966). Higher in the channel the streamlines become less wavy and approach straight lines. Table 2 shows values of the stream function near the critical layer.

In view of the foregoing limited results, the following tentative conclusions can be made:

1) The inverse hyperbolic tangent velocity profile, a result of the theory of the model, can be fitted to almost any known profile type. The profile which has a large value for $-U''$ near the critical level and for which the parameter $L(0)$ has the value 5×10^{-5} produces surface pressures which can be made to converge on the desired value by varying the wall perturbation in l.

2) Wall pressure is extremely sensitive to very small changes in $l_1(0)$ and insensitive to moderate changes in γ.

3) Reasonable wall pressure (and wave growth rates) can be obtained with an appropriate choice of $l_1(0)$. This indicates that small, unequal variations in the surface roughness, such as would be caused by parasitic capillaries, are very important in the wave-generation process.

4) Shear is independent of imaginary values of $l_1(0)$. Therefore, to vary the shear stress, $Re(l_1(0))$, γ or $L(0)$ must be varied individually or in combination as indicated by further analyis of the model.

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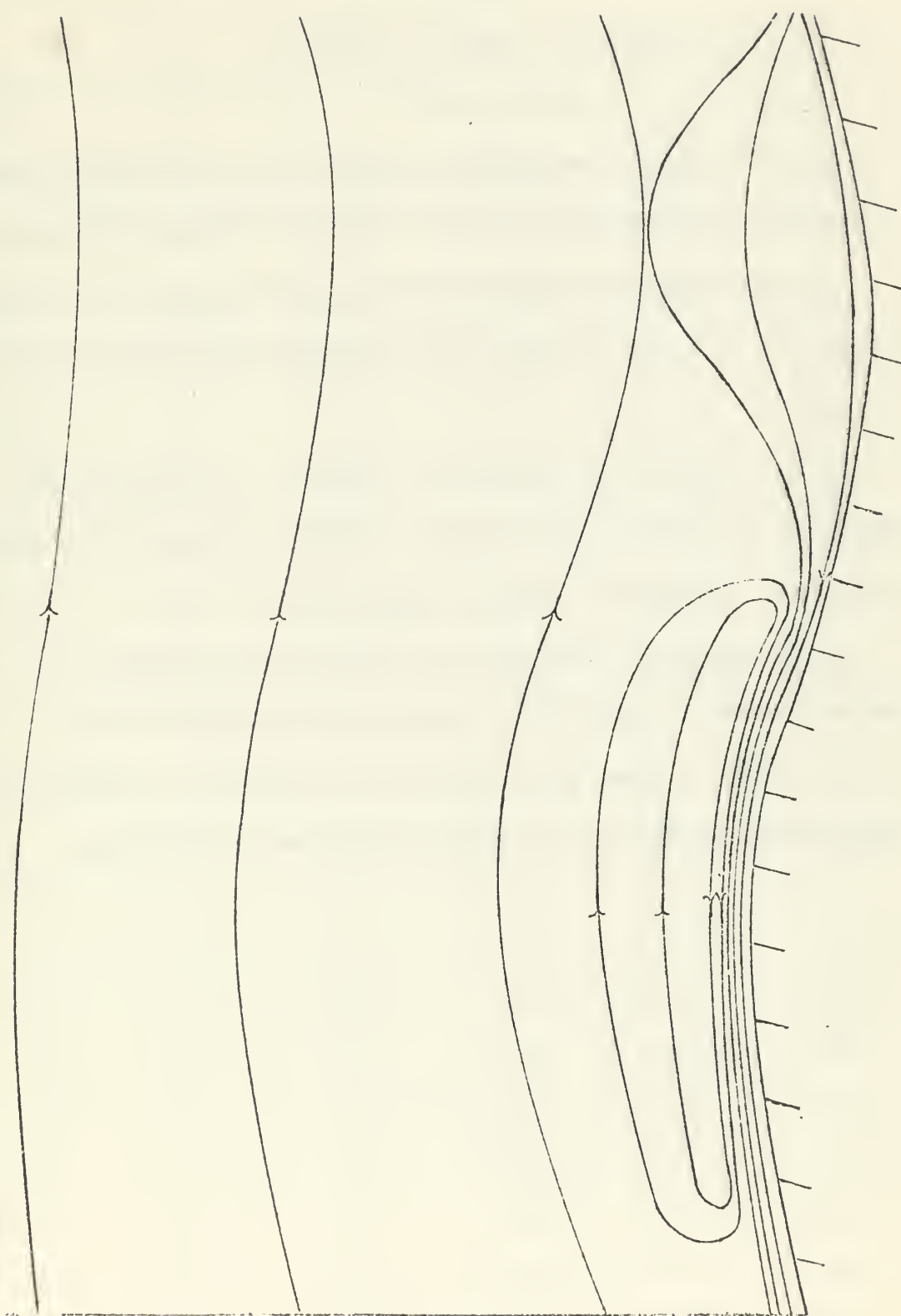
Streamline Pattern

Figure 7.

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Based on the above results, the following suggestions are made:

1) $\llcorner$ (0) should be increased slightly to

$$\llcorner (0) = 10^{-4}$$

in order to reduce the shear stress relative to the wall pressure. One

computer run indicates that the shear stress is halved for this value.

2) Additional wave numbers should be investigated, say, in the

range $5 < k < 20$, and their effect on wall pressure and shear stress

noted.

3) The model should be checked using data from Stanton and

Motzfeld as quoted in Ursel? (1956), and with recently obtained laboratory

data (see, for example, Zagustin, et al., 1966).

In summary, the model appears to be fairly well suited to a

theoretical study of air-stream turbulence and wave generation.

Unfortunately, the long time required for a computer run prevented the

calculation of further data using appropriate parametric values.

$$k\xi$$

η	0.0	0.1	0.2	0.3	0.4
0.0	0.0	0.0	0.0	0.0	0.0
0.002	-1.522	-1.256	0.508	0.434	0.216
0.005	-0.683	-0.571	-0.227	0.218	0.594
0.008	-0.299	-0.253	-0.080	0.154	0.359
0.010	-0.138	-0.116	-0.007	0.145	0.285
0.015	0.110	0.101	0.129	0.183	0.243
0.020	0.244	0.229	0.238	0.269	0.309
0.025	0.325	0.315	0.336	0.381	0.432
0.035	0.424	0.440	0.525	0.646	0.758
0.045	0.506	0.556	0.722	0.940	1.128

$$k\xi$$

η	0.5	0.6	0.7	0.8	0.9
0.0	0.0	0.0	0.0	0.0	0.0
0.002	1.533	1.266	0.518	0.426	-1.522
0.005	0.757	0.645	0.300	-0.145	-0.520
0.008	0.456	0.410	0.236	0.003	-0.202
0.010	0.357	0.336	0.228	0.075	-0.065
0.015	0.284	0.293	0.265	0.211	0.152
0.020	0.343	0.359	0.349	0.319	0.278
0.025	0.470	0.480	0.459	0.414	0.363
0.035	0.817	0.801	0.716	0.594	0.482
0.045	1.212	1.162	0.996	0.778	0.591

Table 2.   Values of the stream function near the critical layer

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```
C   THE EFFECT OF WIND TURBULENCE ON WAVE GENERATION
    IMPLICIT REAL *8 (A-H,O-Z)
    COMPLEX *16 F,ETAK, U, U1, A1, A11P, H1P, E,E1P, G1P, PNEW, YF, GG,
   1FCNG, WK, A2A, TOP, BOT, QUO, TANHI, CON, A3, A3B, DENOM, R1,
   1ELLO, C2, PWALL, G, TT, FF, FR, A, GAMMA, WK2, D, E2P, H2P,
   1G2P, PSI
    DIMENSION G(3,4), YIN(6), FCNF(6), YF(6,4), FCNG(6,4), F(6),
   1S(6,7), X(6), Y(6,4), TT(3), FF(6), PSI(11),
    EP = 1.D-03

C   KK AND KL ARE INDICES FOR STORAGE SUBROUTINE. 7 IS
C   IMAGINARY ONE
    KK = 1
    KL = 49
  5 WRITE (6, 5000) EP
    Z = (0.D0, 1.D0)

C   MATRIX OF INPUT VALUES AT ETA = 0. FOR HOMOGENEOUS EQUATION
    DO 100 I = 1, 6
    DO 100 J = 1, 4
100 Y(I,J) = (0.D0+00, 0.D+00)
    Y(3,2) = (1.D0, 0.D0)
    Y(4,2) = (1.D0, 0.D0)
    Y(6,3) = (1.D0, 0.D0)

C   FROUDE NUMBER
    FR = (1.01800, 0.D0)
C   WAVE NUMBER (DIMENSIONLESS)
    WK = (1.5D0+01, 0.D0)
C   WAVE CELFRITY INPUT (DIMENSIONLESS)
    C2 = C1.D0/(FR * WK)
    C2 = CDSQRT(C2)
    BETA = WK * C
    WRITE (6, 5100) WK, C, BETA, FR
    DO 88 J = 1, 4
    ETA = 0.D0

C   THE FACTORS FOR U1
    ELLO = (5.D-05, 0.D0)
    A2A = 1.D0 + 7.5D0*ELLO
    A2A = 1.D0/CDSQRT(A2)
    TOP = 1.D0 + A2A
    BOT = 1.D0 - A2A
    QUO = TOP/BOT
    TANHI = 0.5D0*CDLOG(QUO)
    CON = (1.5D0*CDSQRT(A2))/TANHI
    A3 = DSQRT(1.D0 - ETA)
    A3B = (1.D0 - ETA)**3
```

```
      DENOM = A2 - A3B
      U1 = (CON*A3)/DENOM
C     INPUT VALUES AT ETA = 0.  FOR INHOMOGENEOUS EQUATION
      Y(1,4) = C
      Y(2,4) = -U1 + BETA
      Y(5,4) = (0.D0, 0.D0)
      DO 66 I = 1, 6
   66 YIN(I) = Y(I,J)
C     EVALUATE FUNCTION F AND L AND THEIR DERIVATIVES
    1 CALL FUNCTF (YIN, J, WK, C, ETA, FCNF, U1, U, EP, H, H1P, E,
     1 E1P, GG, G1P)
      ETAK = ETA*WK
      DO 75 I = 1, 6
   75 YF(I,J) = YIN(I)
      DO 80 I = 1, 6
   80 FCNG(I,J) = FCNF(I)
   88 CONTINUE
  121 CONTINUE
      DO 90 J = 1, 4
      A1 = (C - U)*CDEXP(-ETAK)
      A11P = -U1*CDEXP(-ETAK)
      B1 = (0.D0, 0.D0)
      IF (J - 4) 21, 22, 21
   21 CONTINUE
C     MATRIX INPUT FOR F, F PRIME AND L SOLUTIONS (HOMOGENEOUS)
      G(1,J) = FCNG(1,J)
      G(2,J) = FCNG(2,J)
      G(3,J) = FCNG(5,J)
      GO TO 23
C     MATRIX INPUT FOR F, F PRIME, AND L SOLUTIONS (INHOMOGENEOUS)
   22 G(1,J) = -FCNG(1,J) + A1
      G(2,J) = -FCNG(2,J) + A11P
      G(3,J) = -FCNG(5,J) + B1
   23 CONTINUE
   90 CONTINUE
C     SET UP THE MATRIX
      J = 1
      DO 110 I = 1, 5, 2
      S(I,1) = REAL (G(J,1))
      S(I,3) = REAL (G(J,2))
      S(I,5) = REAL (G(J,3))
      S(I,2) = AIMAG (-G(J,1))
      S(I,4) = AIMAG (-G(J,2))
      S(I,6) = AIMAG (-G(J,3))
  110 J = J + 1
```

```fortran
      J = 1
      DO 115 I = 2, 6, 2
      S(I,3) = AIMAG (G(J,1))
      S(I,5) = AIMAG (G(J,2))
      S(I,7) = AIMAG (G(J,3))
      S(I,2) = REAL (G(J,1))
      S(I,4) = REAL (G(J,2))
      S(I,6) = REAL (G(J,3))
      J = J + 1
      S(1,7) = REAL (G(1,4))
      S(1,7) = AIMAG (G(1,4))
      S(2,7) = REAL (G(2,4))
      S(3,7) = AIMAG (G(2,4))
      S(4,7) = REAL (G(3,4))
      S(5,7) = AIMAG (G(3,4))
      S(6,7) = REAL (G(3,4))
  115 CONTINUE
C SOLVE THE MATRIX FOR THE COMPLEX CONSTANTS TT(1) THROUGH TT(3)
      CALL SUPER(S, X)
      TT(1) = DCMPLX(X(1), X(2))
      TT(2) = DCMPLX(X(3), X(4))
      TT(3) = DCMPLX(X(5), X(6))
      WRITE(6, 5800) (TT(I), I = 1, 3)
      WRITE(6, 5900) DERIVATIVES AT ETA
C EVALUATE F AND ITS DERIVATIVES AT ETA
      DO 74 I = 1, 6
      FF(I) = (0.D0, 0.D0)
      DO 72 M = 1, 3
   72 FF(I) = FF(I) + TT(M)*FCNG(I,M)
      FF(I) = FF(I) + FCNG(I,4)
   74 CONTINUE
C CALL AUXILY (WK, FTA, A, GAMMA, U, U1, U2, U3, U4, ETAK, WK2,
      CALL GG, H, D, GIP, HIP, E1P, G2P, H2P, E2P, A3B)
C EVALUATE THE STREAM PRESSURE
      PNEW = U1*FF(1) - (U - C) *FF(2) - (2.D0*Z)/WK*(H*FF(4)
     1 + HIP*FF(3) + E*FF(6) + E1P*FF(5) + COEXF(-ETAK)*(-GC*WK + G1P))
      WRITE (6,6000) ETA, U1, U2, U3, U4, D, H, GG, E,
     1 HIP, H2P, VGIP, G2P, E1P, E2P, PNEW
C OBTAIN FINAL VALUES FOR F AND L AND THEIR DERIVATIVES
C AT ETA = ETAL
      FTAL = 0.D0
      DO 76 I = 1, 6
      F(I) = (0.D0, 0.D0)
      DO 77 M = 1, 3
   77 F(I) = F(I) + TT(M)*Y(I,M)
      F(I) = F(I) + Y(I,4)
   76 CONTINUE
```

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```
      CALL AUXILY (WK, ETAI, A, GAMMA, U, U1, U2, U3, U4, ETAK, WK2,
     1 E, GG, H, D, GIP, H1P, E1P, G2P, H2P, E2P, A3B)
C     EVALUATE THE PRESSURE AT THE WALL
      PWALL = U1*F(3) + (U - C)*F(2) - (2.D0*Z)/WK*(H*F(4)
     1 + H1P*F(3) + E*F(6) - E1P*F(5) + CDEXP(-ETAK)*(-GG*WK + G1P))
      CALL HOARD (ETA, PNEW, PWALL, WK, KL)
      WRITE (6, 7000) PWALL, PNEW, ETA
      WRITE (6, 5000) EP
      WRITE (6, 5100) WK, C, BETA, FR
      WRITE (6, 6100) ETA
      WRITE (6, 6150) (F(I), I = 1, 6), (FF(I), I = 1, 6)
      PREAL = REAL (PWALL)
      PMAG = AIMAG (PWALL)
C     DETERMINE WAVE GROWTH AND SHEAR STRESS
      CALL GROWTH (WK,C, PREAL, PMAG, FR, BETA, F(3), F(5))
C     STEP EP AND CONTINUE
      EP = EP + 2.D-02
      IF (EP .GT. 1.D0) GO TO 901
      DO 140 J = 1, 4
      DO 145 I = 1, 6
  145 YIN(I) = FCNG(I,J)
C     CONTINUE FUNCTF ITERATION USING THE RUNGE-KUTTA GILL METHOD
      CALL FUNCTF (YIN, J, WK, C, ETA, FCNF, U1, U, EP, H, H1P, E,
     1 E1P, GG, G1P)
      ETAK = ETA*WK
      DO 146 I = 1, 6
  146 YF(I,J) = YIN(I)
      DO 147 I = 1, 6
  147 FCNG(I,J) = FCNF(I)
  140 CONTINUE
      GO TO 121
 5000 FORMAT (1H1, T45, 'SOLUTIONS TO MIXING LENGTH PROBLEM', ///,
     1 T42, 'INVERSE HYPERBOLIC TANGENT VELOCITY PROFILE', ///,
     1 T10, 'VALUES FOR WK, C, BETA, FR', ///,
     1 //, T10, ' EP = 1PD20.10, /// )
 5100 FORMAT (T10, 1P4D20.10, // )
 5800 FORMAT (////,'THE CONSTANTS J1 THROUGH J3', /// )
 5900 FORMAT (////, T10, 1P2D20.10, // )
 6000 FORMAT (////, T10, 1P6D20.10, //, T10, 1P6D20.10, //,
     1 T10, 1P6D20.10, //, T10, 1P2D20.10, /// )
 6100 FORMAT (//, 'FINAL VALUES AT ETA = ', 1P2D20.10, //, T10,
 6150 FORMAT (4X, T10, 1P6D20.10, // )
 7000 FORMAT (///, T10, 'PWALL = ', 1P2D20.10, //, T10,
     1 ' PNEW = ', 1P2D20.10, ' AT ETA = ', 1PD20.10, // )
```

```
C     OBTAIN THE FINAL PRINTOUT OF PERTINENT FUNCTIONS
901   KK=KL
      CALL HOARD (ETA, PNEW, PWALL, KK, KL)
      RETURN
      END

C     EVALUATE F AND L AND THEIR DERIVATIVES AT VARIABLE ETA VALUES
      SUBROUTINE FUNCTF (YIN, J, WK, C, ETA, FCNF, U1, U, EP, H, H1P,
     1EL1P, G, G1P)
      IMPLICIT REAL*8 (A-H, O-Z)
      COMPLEX*16 Y, F, Z, FCNF, H, U, U1, U2, U3, U4, D, F, G, G1P,
     1G2P, E1P, E2P, H1P, H2P, WK, WK2, ETAK, FTAK, FTAK1, ETAK1, ETA2,
     1ELLO, ELLIP, ELL2P, E1P, ELLIP2, ELL2, A, GAMMA, YIN,
     1A3, A3A, A3R, TOP1, BOT1, TANH1, TANH12, TOP, BOT, QIN, DENOM, TANH11, C
      DIMENSION YIN(6), FCNF(6), F(6), EL(3), Y(6)
      NT = 0
      N = 6
      Z = (0.D0, 1.D0)
      DO 5 I = 1, 6
5     Y(I) = YIN(I)
11    CONTINUE
C     VARIABLE STEP SIZE FOR THE RUNGE-KUTTA GILL FUNCTION
      HH = 2.5D-05
      IF(EP.GT. 3.D-01) HH = 2.5D-05
      CALL AUXILY ( WK, ETA, A, GAMMA, U, U1, U2, U3, U4, ETAK, WK2,
     1E, G, H, D, G1P, H1P, E1P, G2P, H2P, E2P, A3R)
C     SIX ORDINARY DIFFERENTIAL EQUATIONS TO BE SOLVED
      F(1) = Y(2)
      F(2) = Y(3)
      F(3) = Y(4)
      EL(1) = Y(6)
      IF(J .EQ. 4) 600, 601, 600
C     HOMOGENEOUS EQUATIONS
600   EL(2) = -Y(5)*(A*E - WK2) - A*H*Y(3)
      F(4) = (1./H)*(Z*WK2.*((U-C)*(Y(3)) - WK2*Y(1)) - U2*Y(1))
     1- 2.*H1P*Y(4) - (H2P + WK2*H)*Y(3)
     1- 2.*EL(2) - 2.*E1P*Y(6) - (E2P + WK2*E)*Y(5))
      GO TO 603
C     INHOMOGENEOUS EQUATIONS
601   EL(2) = -Y(5)*(A*E - WK2) - A*H*Y(3) - GAMMA*D
```

```
      1  - A*CDEXP(-ETAK)*(G - WK*D)
      F(4) = (1./H)*(Z*WK/2.*((U-C)*(Y(3) - WK2*Y(1)) - U2*Y(1))
      1 -2.*H1P*Y(4) - (H2P + WK2*H)*Y(3) - E*EL(2) - 2.*E1P*Y(6)
      1 -(E2P + WK2*E)*Y(5) - CDEXP(-ETAK)*(2.*G*WK2 - 2.*WK*G1P + G2P))
  603 CONTINUE
      F(5) = EL(1)
      F(6) = EL(2)
      S = RKLDEQ(N, Y, F, ETA, HH, NT)
      IF (S - 1.D0) 105, 11, 12
  105 WRITE (6, 5250)
   12 CONTINUE
C  UPPER BOUNDARY CUTOFF
      IF(ETA - EP) 11, 99, 99
   99 CONTINUE
      DO 61 I = 1, N
   61 FCNF(I) = Y(I)
 5250 FORMAT (10X, 'ERROR STOP, S LESS THAN ONE', ///)
 5300 FORMAT (T20, 'INPUT DATA FOR J = ', I2, //, T40,
      1 'Y(1) THROUGH Y(6).', ///)
 5305 FORMAT (4X, 1P6D20.10, //)
 5310 FORMAT (T10, 'VALUE OF A = ', 1P2D20.10, ///, T10,
      1 'VALUE OF ELL0 = ', 1P2D20.10, //)
 5400 FORMAT (////, T20, 'OUTPUT DATA FOR J = ', I2, ///,
      1 T40, 'ETA, U AND FCNF(1) THROUGH FCNF(6).', ///)
 5405 FORMAT (4X, 1P3D20.10, //, 1P6D20.10, //, 4X, 1P6D20.10, ///)
 6000 FORMAT (//, T10, 'EL(2) = ', 1P2D20.10, /)
      WRITE (6, 6000) EL(2)
      RETURN
      END
C
      FUNCTION RKLDEQ (N, Y, F, X, HH, NT)
C  THIS SUBROUTINE EVALUATES THE SIX ORDINARY DIFFERENTIAL EQUATIONS
C  AT SELECTED VALUES OF ETA SPECIFIED IN MAIN PROGRAM
      IMPLICIT REAL*8 (A-H, O-Z)
      COMPLEX *16 Y, F, Q
      DIMENSION Y(1), F(1), Q(25)
      NT = NT + 1
      GO TO (1, 2, 3, 4), NT
    1 DO 11 J = 1, N
   11 Q(J) = (0.0D+00, 0.0D+00)
      A = .5D+00
```

```
      X = X +HH/2.D+00
    2 A = 0.52928932188134525D+00
      GO TO 5
    3 A = 1.7071067811865475D+00
      X = X + HH/2.D+00
      GO TO 5
    4 DO 41 I = 1, N
   41 Y(I) = Y(I) + HH*F(I)/6.D+00 - Q(I)/3.D+00
      NT = 0
      RKLDEQ = 2.
      GO TO 6
    5 DO 51 L = 1, N
      Y(L) = Y(L) + A*(HH*F(L) - Q(L))
   51 Q(L) = 2.*A*HH*F(L) + (1. - 3.*A)*Q(L)
      RKLDEQ = 1.
    6 RETURN
      END


      SUBROUTINE AUXILY (WK, ETA, A, GAMMA, U, U1, U2, U3, U4, ETAK,
     1 WK2, E, G, H, D, GIP, H1P, E1P, G1P, E2P, H2P, G2P, A3R)
C     THIS SUBROUTINE CALCULATES THE VELOCITY PROFILE AND ITS
C     DERIVATIVES PLUS OTHER PERTINENT FUNCTIONS (A-H, O-Z)
      IMPLICIT REAL*8 (A-H, O-Z)
      COMPLEX *16 H, U, U1, U2, U3, U4, D, E, G, G1P, ELL, ELLIP, ELL2P,
     1 G2P, E1P, E2P, H1P, H2P, WK, WK2, ETAK, TANHI, CON, DENOM, TANHI2, TOP, QUO,
     1 ELL2, A, GAMMA, A3R, TOP1, BOT1, ROT1, QUO1, CON, DENOM, TANHI1, BOT, QUO,
     1 A3, A3A, A3R
      ELLO = (5.D-05 + 7.5D+00*ELLO
      A2 = 1.0D+00 + A2A
      A2A = 1.0D+00/CDSQRT(A2)
      TOP = 1.DO + A2A
      BOT = 1.DO - A2A
      QUO = TOP/ROT
      TANHI2 = 0.5D+00*CDLOG(QUO)
      TANHI2 = (TANHI2)*(TANHI)
C     VALUES OF A AND GAMMA
      A = (40.D+00*TANHI2)/A2
      GAMMA = (1.D+01, 0.DO)
      A3 = DSQRT(1.DO - ETA)
      A3R = (1.DO - ETA)**3
```

53

```
      A3A = A3**3
      TOP1 = CDSQRT(A2) + A3A
      BOT1 = CDSQRT(A2) - A3A
      QUO1 = TOP1/BOT1
      TANHI1 = 0.5D0*CDLOG(QUO1)
C     EVALUATE U AND ITS DERIVATIVES
      U = 1.D0 - (TANHI1/TANHI)
      CON = (1.5D0*CDSQRT(A2))/TANHI
      DENOM = A2 - A3B
      U1 = (CON*A3)/DENOM
      U2 = -0.5D0/(A3*DENOM) - (3.D0*CON*A3**5)/(DENOM*DENOM)
      U3 = -CON/(4.D0*A3A*DENOM) + (1.5D0*CON*A3A)/(DENOM*DENOM)
     1    + (7.5D0*CON*A3A)/(DENOM**3) + (18.D0*CON*A3**9)/(DENOM**3)
      U4 = -CON*(0.5D0*(2.D0/DENOM)*A3 - (1.5D0/DENOM)*A3 +
     1    (4.5D0/DENOM*DENOM)*A3 - (1.D0/(A3**5)) - 3.D0*(-3.75D0*A3*
     1    (0.75D0/DENOM)*(1.D0/(A3**7)) - 15.D0*(1.D0/DENOM**3)
     1    -27.D0*(A3**7)*(1.D0/(A3**7)) - 54.D0*(A3**13)*(1.D0/DENOM**4)))
      ETAK = ETA*WK
      WK2 = WK*WK
      ELL2 = (A2 - (1.0D+00 - ETA)**3)/7.5D+00
      ELL = ELL*ELL
      ELLIP = (3.0D+00*(1.0D+00 - ETA)*(1.0D+00 - ETA))/7.5D+00
      ELL2P = (-6.0D+00*(1.0D+00 - ETA))/7.5D+00
      E = ELL2*U1*U1
      G = ELL2*U1*U2
      H = ELL2*U1*U1
      D = ELL2*U2*U2
      G1P = 2.*ELL*ELLIP*U1*U2 + ELL2*U2*U2 + ELL2*U1*U3
      E1P = 2.*U1*U2*ELL + U1*U1*ELLIP
      H1P = 2.*ELL*ELLIP*U1 + ELL2*U2
      G2P = 2.*U1*U2*(ELLIP*ELLIP + ELL*ELL2P) + 4.*ELL*ELLIP*U2*U2
     1    + 4.*ELL*ELLIP*U1*U3 + 3.*ELL2*U2*U3 + ELL2*U1*U4
      H2P = 2.*ELLIP*ELLIP*U2 + 2.*ELL*ELL2P*U2 + 2.*ELL*FLL2P*U1
      E2P = 2.*U2*U2*ELL + 4.*U1*U3*ELL + 4.*U1*U2*ELLIP + U1*U1*FLL2P
      RETURN
      END
```

C   SUBROUTINE SUPER (A,X)

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```
C   MODIFIED JORDAN ELIMINATION METHOD
    IMPLICIT REAL *8 (A-H, O-Z)
    DIMENSION A(6,7), X(6)
    K = 7
    N = 6
11  IF(K - I) 13, 6, 15
15  D = 0.D0
    DO 2 I = 2, K
    IF (DABS(A(I-1, 1)) - D) 2, 2, 3
3   L = I
    D = DABS(A(L, 1))
2   CONTINUE
4   IF(L - 1) 5, 6, 5
5   DO 7 J = 1, K
    D = A(L, J)
    A(L, J) = A(1, J)
7   A(1, J) = D
6   DO 8 I = 1, N
8   X(I) = A(I, 1)/X(1)
    IF(K - 1) 12, 13, 12
12  DO 10 J = 2, K
    DO 9 I = 2, N
9   A(I-1, J-1) = A(I, J) - X(I)*D
10  A(N, J-1) = D
    K = K-1
    GO TO 11
13  CONTINUE
    RETURN
    END
```

```
C   SUBROUTINE GROWTH (WK, C, PREAL, P, FR, BETA, F3, F5)
C   THIS SUBROUTINE COMPUTES SHEAR STRESS, AND WAVE GROWTH AS A
C   FUNCTION OF BOTH NORMAL PRESSURE AND NORMAL PRESSURE PLUS SHEAR STRES
    IMPLICIT REAL *8 (A-H, O-Z)
    COMPLEX *16 WK, C, A, DELTA, DA, FR, BETA, FAC, F3, F5, ELLO,
    1 DAT, TAU
    A = (1.D-01, 0.D0)
    ELLO = (5.D-05, 0.D0)
    RHOA = 1.293D-03
    RHOW = 1.027D0
```

```fortran
      ETA = 0.D0
      FAC = 0.5D0 * FR * (RHOA/RHOW) * BETA
      DA = A * FAC * P
      CALL AUXILY (WK, ETA, AA, GAMMA, U, U1, U2, U3, U4, ETAK, WK2,
     1 E, G, H, D, G1P, H1P, E1P, G2P, H2P, E2P, A3B)
      TAU = 2.D0*ELLO*U1*(U1*F5 + ELLO*(U2 + F3))
      TREAL = REAL (TAU)
      DIV = P/PREAL
      ANGLE = DATAN (DIV)
      ANGLE = ANGLE * 57.2957800
      WRITE(6, 6200) ANGLE, TREAL, DA
 100  FORMAT (//, T10, 'PHASE ANGLE =', 1PD20.10, //,
     1 T10, 'SHEAR STRESS =', 1PD20.10, //, T10, 'LINEAR WAVE GROWTH RAT
     1E =', 1P2D20.10, //)
6200  RETURN
      END


C     SUBROUTINE HOARD (ETA, PNEW, PWALL, KK, KL)
C     THIS SUBROUTINE STORES THE VALUES OF WALL PRESSURE AND STREAM PRESSUR
      FOR THE FINAL PRINTOUT
      IMPLICIT REAL * 8 (A-H, O-Z)
      COMPLEX * 16 PNEW, PWALL, PN, PW
      DIMENSION PN(49), PW(49), Y(49)
      IF (KK - KL) 10, 100, 100
 10   PN(KK) = PNEW
      PW(KK) = PWALL
      Y(KK) = ETA
      KK = KK+1
      GO TO 901
 100  WRITE (6, 1000)
      DO 110 J = 1,100
 110  WRITE(6, 1100) Y(J), PN(J), PW(J)
 1000 FORMAT (1H1, T15, 'ETA', T35, 'RPNEW', T55, 'IPNEW', T75,
     1 'RPWALL', T95, 'IPWALL', ///)
 1100 FORMAT ( 4X, 1P5D20.10)
 901  RETURN
      END
```

C SOLUTIONS TO WAVE GENERATION BY TURBULENT WIND FOR THE
C FOLLOWING PARAMETER VALUES

GAMMA = 10.0
L(0) = 0.00005
PERTURBATION IN L (AT ETA = ZERO) = 0.65788
WAVE NUMBER = 15.0

VALUES FOR NORMAL PRESSURE, TANGENTIAL SHEAR STRESS AND WAVE
GROWTH RATE

ETA	PWALL	SHEAR STRESS	GROWTH RATE
0.00	2748.86	29.815	0.6762
0.02	324.94	19.184	0.0790
0.04	352.10	19.185	0.0866
0.06	364.16	19.185	0.0895
0.08	363.97	19.187	0.0895
0.10	350.15	19.187	0.0861
0.12	318.46	19.186	0.0783
0.14	274.37	19.186	0.0675
0.16	236.28	19.186	0.0581
0.18	207.85	19.186	0.0511
0.20	182.64	19.186	0.0499
0.22	157.42	19.186	0.0387
0.24	132.13	19.186	0.0325
0.26	108.01	19.186	0.0266
0.28	86.63	19.186	0.0213
0.30	71.66	19.186	0.0176
0.32	59.69	19.186	0.0147
0.34	49.55	19.186	0.0122
0.36	39.99	19.186	0.0098
0.38	30.50	19.186	0.0075
0.40	21.47	19.186	0.0053
0.42	13.75	19.186	0.0034
0.44	8.13	19.186	0.0020
0.46	4.92	19.186	0.0012
0.48	3.83	19.186	0.0009
0.50	4.13	19.186	0.0010
0.52	5.03	19.186	0.0012
0.54	6.32	19.186	0.0015
0.56	7.32	19.186	0.0018
0.58	9.14	19.186	0.0022
0.60	12.05	19.186	0.0030
0.62	16.39	19.186	0.0040

ETA	PWALL	SHEAR STRESS	GROWTH RATE
0.64	22.19	19.186	0.0055
0.66	29.26	19.186	0.0072
0.68	37.28	19.186	0.0092
0.70	45.99	19.186	0.0113
0.72	55.27	19.186	0.0136
0.74	65.14	19.186	0.0160
0.76	75.70	19.186	0.0186
0.78	87.02	19.186	0.0214
0.80	99.15	19.186	0.0244
0.82	112.07	19.186	0.0276
0.84	125.74	19.186	0.0309
0.86	140.20	19.186	0.0342
0.88	155.54	19.186	0.0383
0.90	171.99	19.186	0.0423
0.92	189.95	19.186	0.0467
0.94	210.24	19.186	0.0517
0.96	234.88	19.186	0.0578
0.98	271.49	19.186	0.0668

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13. ABSTRACT

The paper is concerned with a theoretical study of shearing flow bounded by a wavy surface with consideration of turbulent flow above the air-sea interface. Account is to be taken of turbulence in the flow through the use of Reynolds stresses associated with turbulent flow. An adaptation of the mixing length theory as applied to pipe flow is made for channel flow and the resulting mixing length versus energy relationship is incorporated in the Reynolds stress term. Finally, the rate of wave growth is calculated from the normal surface pressure in phase with the wave slope.

A cirvilinear coordinate system which follows with the wave train is used in order to simplify the formulation of the problem. All parameters are non-dimensionalized and the analysis is made considering a velocity profile adapted from pipe to channel flow.

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	WAVE GENERATION						
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